Three-boson relativistic bound states with zero-range two-body interaction

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Zero-range two-body interaction, applied to the nonrelativistic three-body system, results in the Thomas collapse [1]. The latter means that the three-body binding energy tends to $-\infty$ when the interaction radius tends to zero. Relativistic three-body calculations with zero-range interaction, performed in a minimal relativistic model [2] and in the framework of the Light-Front Dynamics [3], have shown that, due to relativistic repulsion, the three-body binding energy remains finite and the Thomas collapse is consequently avoided.

We have considered [4] in the field-theoretical framework, the problem of three equal mass (m) bosons interacting via zero-range forces. Few points are worth to be precised.

(i) In quantum field theory the number of particles is not conserved. The three-body relativistic system is, therefore, an idealization. Our aim is to study whether this idealized relativistic system still exists for zero range interaction.

(ii) The vacuum remains bare (no vacuum fluctuations) only in the Light-Front Dynamics. That is why we work in this approach.

(iii) In field theory all elementary interactions are point-like and, in this sens zero-range. We consider the Hamiltonian $H_{int} = \lambda\varphi^4(x)$ restricted to two-body system. It generates a two-body kernel which is simply the constant λ. This is the precise meaning of zero-range two-body interaction used in our study.

A relativistic Faddeev equation is derived, which provides a parameter-free relation between the two-body bound state mass M_2 and the three-body one M_3. It coincides with equation (11) from [3] except for the integration limits.

In figure 1(a) our calculations (solid line) of the three-body ground state mass M_3 as a function of the two-body one M_2 together with the dissociation threshold $M_3 = M_2 + m$ are displayed (in m units). In the zero two-body binding limit $B_2 = 2m - M_2 \to 0$ the three-boson system has a binding energy

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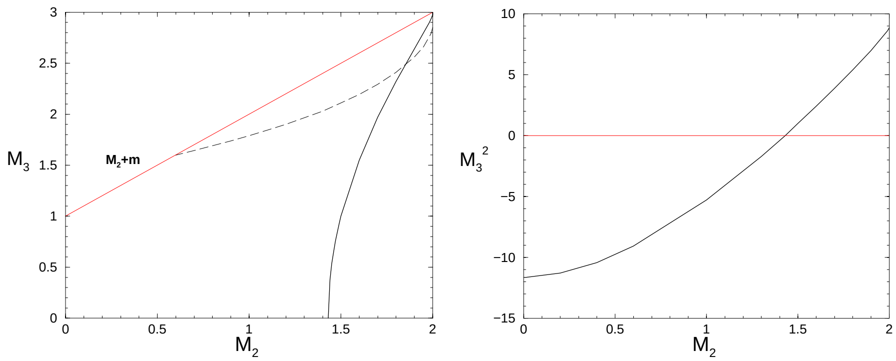
$B_3 \approx 0.012\,m$. Dash line is the result obtained using the integration limits



(a) (b)

Figure 1. (a) Three-body bound state mass M_3 v.s. the two-body one M_2. Solid line corresponds to present work, dash line and dots – to calculations using the integration limits of [3]. (b) Three-body bound state mass squared M_3^2 v.s. M_2.

of [3]. In both cases, like in [2], the repulsive relativistic effects produce a natural cutoff, leading to a finite spectrum and – in the Thomas sense – an absence of collapse. However, the behaviours of the solid and dashed curves qualitatively differ from each other. When M_2 decreases, we found that the three-body mass M_3 (solid line) decreases very quickly and vanishes at the two-body mass value $M_2 = M_c \approx 1.43\,m$.

Results for M_3^2 are given in figure 1(b). For $M_2 \leq M_c$, M_3^2 becomes negative and there are no physical solutions with real M_3 mass. However M_3^2 remains finite in all the two-body mass range $M_2 \in [0,2]$ with $M_3^2 \approx -(3.41\,m)^2$ in the limit $M_2 \to 0$.

We conclude that in the system of three relativistic bosons with zero-range interaction, a relativistic counterpart of Thomas collapse takes place for strong enough two-body forces such that $M_2 < M_c$. If in nonrelativistic treatment the system ceases to exist when its binding energy tends to $-\infty$, in the relativistic approach the system does not exist when its mass squared becomes negative.

References

1. L.H. Thomas, Phys. Rev. **47**, 903 (1935).
2. J.V. Lindesay and H.P. Noyes, Preprint SLAC-PUB-2932 (1986).
3. T. Frederico, Phys. Lett. **B282**, 409 (1992).
4. J. Carbonell, V.A. Karmanov, nucl-th/0207073 (submitted for publication).